Math Review

Experiment P01

Time Required for Completion: Approximately 75 minutes
Lab Report Grading Guide: Introduction (*0 pts*), Data (*0 pts*), Data Analysis [Tables (*42 pts*) & Analysis Questions (*58 pts*)], Conclusion (*0 pts*).

OBJECTIVES

- To review the following selected mathematical topics:
  - SI Units & the Metric System
  - Significant Figures
  - Importance of Units
  - Uncertainty in Measurement
  - Graphical Analysis
  - Scientific Notation & the Powers of Ten
  - Estimating Order of Magnitude
  - Percent Difference
  - Systematic Uncertainties
  - Linear Functions
- To solve a variety of mathematical problems.

EQUIPMENT NEEDED

Experiment Apparatus:
Ruler (**Student provide**)
Graph Paper (**Student provide**)
Calculator (scientific) (**Student provide**)

THEORY

SI Units & the Metric System

In science we use a standardized method of units and measurement called the SI (International System) which is derived from the metric system. This system, due to its simple and logical nature, is used because the fundamental physical quantities of **length (meter)**, **mass (kilogram)**, **time (second)** and **temperature (degree Celsius or Kelvin)** may be easily derived by either multiplying or dividing by an appropriate powers of ten.

These multiples and submultiples of the physical quantities are denoted by the prefixes attached to the units. The prefixes reflect the power of ten involved and may be used to readily denote a change in size of both large and small quantities. For most of our experimental purposes in Physics 111N/226N/231N, the most commonly encountered prefixes are mega, kilo, deci, centi, and milli.

Table of Prefixes

Prefix	Symbol	Signifies	Power of Ten
tera	T	1,000,000,000,000	10^{12}
giga	G	1,000,000,000	10^{9}
mega	M	1,000,000	10^{6}
kilo	k	1,000	10^{3}
hecto	h	100	10^{2}
deka	da	10	10^{1}
Base Unit: Meter (m), Gram (g), Second (s)			10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	µ	0.000001	10^{-6}
nano	n	0.000000001	10^{-9}
pico	p	0.000000000001	10^{-12}

Converting between units is very easy. Move the decimal point either right or left a specific number of places, and change the prefix to reflect the new powers of 10 associated with the unit.

In example, by **moving the decimal point to the right**:

1.0 km = 1000.0 m or **1.0 m = 100.0 cm = 1000.0 mm**

Likewise, by **moving the decimal point to the left**:

1.0 mm = 0.1 dm = 0.01 cm = 0.001 m

Scientific Notation & the Powers of 10

Scientific notation and the powers of 10 notation are both compact, shorthand methods of writing very large or small numbers. In fact, scientific notation follows the basic rules of powers of 10 notation (expressing a number as a product of two numbers), with only a slight modification of how the result is expressed, in that:

In power of 10 notation, the first part of the result **may be expressed as equal to or greater than 1**, and the second part of the result is expressed as 10 raised to a power.

In scientific notation, the first part of the result is **always expressed as greater than or equal to 1, but less than 10**, and the second part of the number is expressed as 10 raised to a power.

For example: consider the number of molecules in one liter of air. To write out the number of molecules, approximately 780000000000000000000000, requires 78 followed by 22 zeros.

Expressed in powers of 10 notation we could write this out as 78×10^{22}, or 780×10^{21}, or even 7800×10^{20}, etc., all of which would be correct powers of 10 notations.

However to be shown correctly in scientific notation, this number would be written out **only** as 7.8×10^{23}. (*Note that 7.8 is greater than 1, but less than ten*).

To convert large numbers to **powers of 10**, start at the decimal point (or where the decimal point is understood to belong) and count to the left. The number of digits is the power of ten. In our example the number of molecules was 780000000000000000000000 with the decimal understood to be after the last zero.

Starting at the decimal, and counting to the left, there are 22 zeros so:

780,000,000,000,000,000,000,000 = $\mathbf{78 \times 10^{22}}$ molecules.

However, remember that in scientific notation the rule is that the first numbers are shown as greater than one, but less than ten, so we must move the decimal point one more space to the left, (increasing the exponent by one), resulting in the **correct scientific notation** of: $\mathbf{7.8 \times 10^{23}}$ molecules.

Small numbers are written in powers of 10 and scientific notation by using **negative exponents**. The negative exponent reflects the reciprocal of the power indicated. Then deduce the correct power of 10 by starting at the decimal point and counting to the right to the new point where the decimal will be located. The number of spaces is the (negative) power of 10.

Example:

0.0005 = $\mathbf{5 \times 10^{-4}}$ because we ***moved the decimal point 4 places to the right***.

Thus,0.000000000023 = $\mathbf{2.3 \times 10^{-11}}$. Note that we didn't move the decimal all the way to the right because if we had done so, we would have violated the rule of having the first part of the scientific notation being greater than 10.

<u>Rule</u>: If the decimal is moved to the **left**, the **power of 10 is increased** by the number of integers moved, and if the decimal is moved to the **right**, the **power of 10 is decreased** by the number of integers moved.

Scientific notation also eases mathematical operations. Imagine how tedious it would be to multiply the two very large numbers. However, by using powers of 10 and scientific notation it is easily accomplished.

Example: We want to multiply 780000000000000000000000 by 90000000000000000000000.

To **multiply the numbers**, we first convert the numbers to scientific notation, resulting in $7.8 \times 10^{23} \times 9 \times 10^{22}$. Next, we simply multiply the first part 7.8×9 and then for the second part, we **add** the powers of 10 exponents, $10^{23} + 10^{22}$. The result is 70.2×10^{45}. However, this result violates the scientific notation rule of expression, so we move the decimal one place to the left and increase the exponent by one, giving us the correct scientific notation result of 7.02×10^{46}.

$$7.8 \times 10^{23} \times 9 \times 10^{22} = (7.8 \times 9 = 70.2) \times (10^{23} + 10^{22} = 10^{45}) = 70.2 \times 10^{45} = \mathbf{7.02 \times 10^{46}}$$

To **divide numbers**, the process is the same except that you **subtract** the exponents.

To **add or subtract two numbers** expressed in powers of 10 and scientific notation, **the powers of ten must be the same for each number**.

For example: We want to add 7.8×10^{23} and 9×10^{22}.

First, express both numbers in the same power of ten. In this case, we can express 7.8×10^{23} as 78×10^{22}. This allows us to simply add the first part (integers) of the number and do nothing to the second part (exponents) as they both already reflect the same power of 10.

The power of 10 notation result is 87×10^{22} but we must express this result in correct scientific notation. To do so, we move the decimal one place to the left and add one to the exponent, resulting in a correctly expressed scientific notation answer of, $\mathbf{8.7 \times 10^{23}}$.

$$7.8 \times 10^{23} + 9 \times 10^{22} = 78 \times 10^{22} + 9 \times 10^{22} = 87 \times 10^{22} = \mathbf{8.7 \times 10^{23}}$$

Remember the following three rules when doing mathematical operations:

1. **To <u>multiply</u>:** **Multiply the integers and add the exponents**
2. **To <u>divide</u>:** **Divide the integers and subtract the exponents**
3. **To <u>add or subtract</u>:** **The numbers must have the same powers of 10. When they have the same power of 10, add or subtract only the integers.**

Significant Figures

No measurement is perfectly accurate; there is always some uncertainty in a measured quantity. However, <u>you can roughly indicate the accuracy of a measurement by carefully choosing how many digits are used to represent a number</u>.

For example, if you measure the length of something with a ruler marked in millimeters (mm), you can estimate the length between the millimeter markings. If you measure the length to be 34.8 mm, the first two figures, 34, are the *sure* digits, and the 8 is the *estimated* or *doubtful* or *uncertain* figure. <u>Measurements are recorded using all sure figures plus one estimated figure</u>. **In calculations, all figures up to and including the <u>first</u> doubtful one are called *significant* figures.**

The **decimal point** *fixes the magnitude of a measurement but does not indicate the number of significant figures*. Thus a result, whose digits are 265, where the last digit is uncertain, is said to have three significant figures. This would be the case whether the number happened to be 0.000265 or 2.65×10^4 or 26.5.

Recording a length as 5.2 cm means that the actual length must lay between 5.15 and 5.25. Thus the uncertainty is 0.1 part in 5.2 giving a percentage uncertainty of 2% (0.1 ÷ 5.2 × 100%). Recording the length as 5.20 cm means that the length lays between 5.195 and 5.205. This number is more accurate, with an uncertainty of 0.01 part in 5.20 or 0.2%.

In calculations you must be careful about the number of significant figures in the result. The following are a few simple working rules:

1. **In addition or subtraction,** drop every digit in the result that falls under a column containing a non-significant figure.

   Example:

   ```
          2.65
      42841
   +     50.243
      42893.893 = 42894
   ```

2. **In a product or quotient,** retain no more figures in the result than are contained in the quantity having the **fewest** significant figures.

   Example: (37.4) (352.76) = 13193.224 = 1.32×10^4
   ↑
   Limiting term
   has three significant figures

3. **In dropping figures that are not significant,** the last figure retained should be increased by one if the first figure dropped is 5 or more. (Use rounding rules).

4. **Numbers should be expressed in scientific notation whenever confusion might arise.** Suppose a tabletop measured 54.8 cm by 37.5 cm. Its area should not be written as 2055 cm^2, which implies an uncertainty of only one part in 2055. The area should be expressed in scientific notation as 2.06×10^3 cm^2. The correct uncertainty of one part in 206 is now indicated.

5. **Leading zeros are not significant.** 0.0025 may be written as 2.5×10^{-3}

6. **Trailing zeros *after* the decimal point are significant.** 1.20 has three significant figures. **However, trailing zeros *before* the decimal point are more ambiguous.** For example, the number 2000 may have 1, 2 or 3 significant figures. **If all trailing zeros are significant, then a decimal point is included in the number.**

   In example, by adding a decimal point, the number 200., the number clearly has three significant digits, and likewise, the number 20. clearly has two significant digits. The only sure method to avoid ambiguity in indicating the significance of trailing zeros is to write the number in scientific notation ($2. \times 10^3$, 2.0×10^3, or 2.00×10^3).

Order of Magnitude Estimate

Using scientific notation it is much easier to distinguish between groups of very large or groups of very small numbers. For example, without tediously counting the zeros, which of the following numbers is bigger, can you easily tell which of the following numbers is larger? 780000000000000000000000 or 9000000000000000000000? However, if these numbers are written in scientific notation as, 7.8×10^{23} and 9×10^{22}, then it becomes readily apparent which one is larger, because one number is 10^{22} power and the other is 10^{23} power.

Yet, when dealing with very large or very small numbers, it is sometimes simply sufficient to know the power of ten associated with the quantity. For example, the number of molecules in a cubic centimeter of air is approximately 3×10^{19} molecules. If this number is rounded to the nearest power of ten (10^{19} molecules), this is called an **order of magnitude estimate**. This conveys the immense magnitude of this quantity, which is 1 followed by 19 zeros. In our galaxy there are approximately 10^{10} stars, as an order of magnitude estimate. In some situations it does not matter whether there are actually 13 billion stars or 7 billion stars — the order of magnitude estimate conveys the necessary information.

Importance of Units

Various physical quantities will be measured and calculated during laboratory experiments. Since these are physical quantities, every measurement results in a number and some type of physical unit. If the length of a piece of notebook paper is measured, then the quantity is recorded as 27.7 cm or 0.277 m. The annotation of **cm** or **m** represents the units in which the length is measured. **Note: It is incorrect to record a physical quantity without recording the units**. For example, using the formula **Area = length × width**, the calculation of the area of a piece of notebook paper should be written:

$$27.2 \text{ cm} \times 21.3 \text{ cm} = \mathbf{590\ cm^2}$$

Keeping track of the units in each calculation will make it easier to detect errors such as dividing when you should have multiplied.

Percentage Difference

After obtaining a result from a measurement or a calculation, it is frequently desirable to compare the value obtained with a standard value. The aim is to see how closely the value obtained agrees with the standard value. For example, suppose a floor tile should measure 25 cm on a side. This is the accepted standard value. However, your measurement of the tile yields a value of 24.5 cm. The difference between the measured value and the standard value is 0.5 cm. Yet, this difference does not actually tell how "good" the measurement is.

To see why this is so, consider the distance between markers on the highway that are 1 km apart. Suppose the distance between the markers is measured as 1000.005 m. The difference between the standard value and measured value is 0.5 cm, the same as for the floor tile example. However the number of significant figures in the latter measurement indicates that the second measurement is "better" (i.e., more accurate) than the first.

One method of comparing two numbers is the **percentage difference**. In the example below the % difference between the accepted (standard) value and the measured value is calculated:

$$\%\ difference = \left| \frac{accepted\ value - measured\ value}{accepted\ value} \right| \times 100$$

Thus for the tile measurement, the percentage difference between the standard value and the measured value is:

$$\%\ difference = \left| \frac{25\ cm - 24.5\ cm}{25\ cm} \right| \times 100 = 2\ \%$$

It should be noted that the percentage difference is recorded as the absolute value of the number and thus, the result is always a positive number regardless of whether the measured value is greater or less than the standard value. For the distance between the highway markers the percentage difference is:

$$\% \ difference = \left| \frac{1000\ m - 1000.005\ m}{1000\ m} \right| \times 100 = 0.0005\ \%$$

Comparing the two percentage differences shows that the measurement of the distance between the highway markers is a "better" (more accurate) measurement (smaller % diff).

Uncertainty in Measurement

No physical measurement can be perfectly accurate. The accuracy of a measurement is limited by the method and equipment used to perform the measurement, as well as the skill of the experimenter. As previously mentioned, the accuracy of a measurement is reflected in the number of significant figures used to express the number. However, the *number of significant figures in a value does not always accurately convey the uncertainty in a measurement*. For instance if the length of a rod is written as 12.52 cm we would assume that the true length lies between 12.51 and 12.53 cm, based on the number of significant figures.

But what if we were to measure the length of the rod several times and find that it is somewhere between 12.49 and 12.55? Then we should express the length as 12.52 ± 0.03; and the number of significant figures is no longer enough information to represent the accuracy of the measurement. In this example, the length of the rod is measured to be 12.52 and the uncertainty in that measurement is 0.03.

The uncertainty in an experimental result is merely an *estimate* and as such is only quoted to one significant figure. It would not be correct to write the length of a rod as 12.52 ± 0.036 because there are two significant figures in the uncertainty. Furthermore, **the measured quantity should not be written more accurately than the uncertainty**. If the uncertainty in the rod is 0.2, the length of the rod should be expressed as 12.5 ± 0.2 cm. Both the length and the uncertainty are quoted in the same decimal place. See Introduction 3 of this lab manual for a more detailed explanation of how to calculate uncertainties.

Systematic Uncertainties

The measurement uncertainties considered so far are essentially random in that they are equally probable in each direction.

There are other sources of uncertainty that are not random. These are referred to as systematic uncertainties (or systematic errors). For example if a micrometer does not read zero when the jaws are closed, all readings will be too large or too small; if a barometer vacuum is imperfect every pressure reading will be too small; if a meter is not calibrated accurately readings taken from it will be erroneous due to the inaccurate calibration as well as due to the uncertainty in the instrument reading.

These errors should be eliminated as far as possible. For example, one can allow for the zero error of the micrometer, make a correction for the imperfect vacuum in the barometer or calibrate the meter. If they cannot be eliminated then these uncertainties should be noted with the final result.

Graphical Analysis

In many laboratory experiments a set of data is analyzed with the aid of graphs. Given below are a few rules for graph plotting.

1. A title must be written at the top of the page summarizing the information described by the graph.

2. Each axis should be labeled with the quantity being plotted showing the units. At regular intervals along the axis indicate the value of the coordinate. It is customary to plot the quantity that is arbitrarily varied (the independent variable) along the horizontal axis (abscissa) and the dependent quantity (the dependent variable) along the vertical axis (ordinate). Scale readings do not necessarily always start with zero.

3. Use a hard pencil to plot each point as a small, clear dot, enclosed by a circle. If more than one curve is plotted on the same graph, enclose the plotted points on the additional curves with squares, triangles, etc. Identify each curve in a key on the graph legend.

4. Pencil in a curve that will show the variation of the quantities represented. Disregard obviously erratic points and draw a smooth curve so that about the same number of points lie on each side of it. Do not attempt, by using a wavy or zigzag line, to strike all points.

5. Plot the graph on as large an area of the graph paper as possible.

Linear Functions

The graphical representation of the equation $y = mx + b$, where *x* is the independent variable, *y* is the dependent variable and *m* and *b* are constants, is a straight line of slope *m* and intercept on the vertical axis *b*. It is often necessary to determine the slope of a straight line. This is done by taking the coordinates of two *distant* points and using the equation,

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The distances, Δx and Δy should be shown on the graph as two dotted lines comprising the adjacent sides of a right triangle. Where appropriate, the *x* and *y* intercepts should be given.

Figure P00.1 is an example of a graph plotted from the results of the velocity of a freely falling body as a function of time. The results are expected to follow the equation of a uniformly accelerating body, $v = at + v_0$, where v_0 is the initial velocity at time $t = 0$ and the acceleration *a* is due to gravity. **The acceleration is determined from the slope of the line of best fit to the experimental data,** i.e.:

$$a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a = \frac{13.25 - 3.40}{(60. - 10.)/40} \frac{m/s}{s}$$

$$a = 9.85\, m/s^2$$

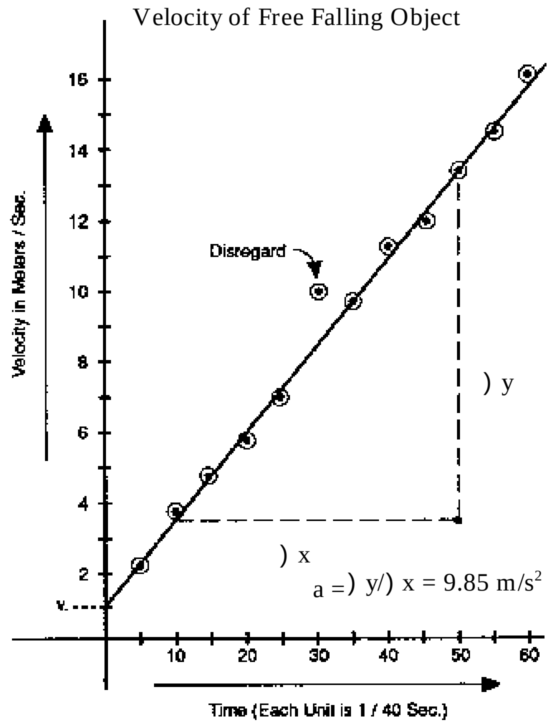

Figure P01.1: Graphing

Any equation which can be simplified to the form y = mx + b can be plotted as a straight line which is easily analyzed. *Example:* $y = kx^2 + b$

This can be compared to the accepted value of the acceleration due to gravity of g = 9.81 m/s^2. The initial velocity of the body is v_0 = 0.94 m/s, shown as the y-intercept on the graph in Figure P01.1.

Plotting y against x will yield a parabolic curve from which k cannot be easily determined. Let X / x^2, so that $y = kX + b$. In this case, plotting y vs X (*i.e.*, y vs x^2), will yield a straight line, the slope k of which can now be easily calculated.

PROCEDURE

1. Complete the Experiment P01 laboratory report.

Old Dominion University
Physics 111N/226N/231N Laboratory Report

Experiment P01:
Math Review

____
Student Name

Lab Information

____________________ Lab Section (Day/Time)	____________________ Date of Lab
____________________ Lab Instructor	____________________ Date Submitted

INTRODUCTION (An introduction is not required for this experiment)

DATA (A data section is not required for this experiment)

DATA ANALYSIS (Complete the tables below. A narrative section is not required for this experiment.)

1. (*6 pts*). Complete the following table **by filling in the missing values**.

Prefix	Symbol	Signifies	Power of 10		Prefix	Symbol	Signifies	Power of 10
milli	m	0.001			kilo	k		10^{3}
	n	0.000000001	10^{-9}		mega	M	1,000,000	
centi		0.01	10^{-2}			G	1,000,000,000	10^{9}

2. (*20 pts*). Complete the table **by showing the quantities in the metric units indicated**. (You may use power of 10 notation as desired.)

Quantities	Shown in nm	Shown in mm	Shown in cm	Shown in m	Shown in km
265 nm					
35.2 km					
101.36 cm					
1200 mm					
6.23 m					

3. (*8 pts*). **Round off** the following numbers to **three significant figures**. (Do not use power of 10 notation.)

	Value	Result		Value	Result
a.	**6.22493 =**		**e.**	**710.005 =**	
b.	**256.14 =**		**f.**	**0.0031462 =**	
c.	**0.61254**		**g.**	**28.1544 =**	
d.	**130,590,281.60 =**		**h.**	**10.55 =**	

4. (*8 pts*). Perform the **indicated operation** and **express the answer in both powers of 10 and scientific notation** to the **correct number of significant figures**.

	Operation	Powers of 10 Notation	Scientific Notation
a.	$(0.314) \times (0.006313 \times 10^{6}) =$		
b.	$(1.11265 \times 10^{-13}) + (2.22 \times 10^{-16}) =$		
c.	$(3.3456 \times 10^{20}) \div (2.244 \times 10^{25}) =$		
d.	$(1.999 \times 10^{6}) \times (1.999 \times 10^{9}) =$		

DATA ATTACHMENTS (staple to the back of your lab report)

► Analysis question 4 Velocity Graph.

ANALYSIS QUESTIONS (*58 pts possible*)

1. (*8 pts*). Calculate the percentage difference (% difference) for the following case:

During their experiment, the lab partners conducted several trials to determine the value of acceleration of a coffee filter dropped in free-fall from a height of 2 meters and they found that the average value of acceleration was 9.78 m/s^2. However their reference manual states that the accepted value of free-fall acceleration is 9.81 m/s^2. Find the percentage difference between the accepted value and the measured value. Show your work.

2. (*15 pts*). Find the uncertainty in the following case:

In a physics ballistics experiment using a 25 mm steel ball projectile fired from a spring-loaded launcher, the lab partners carefully measured the distance from the launcher muzzle to the projectile impact point. They conducted several trials and collected the data shown in the table below. Based on the collected data, determine the uncertainty associated with the distance that the launcher can fire the projectile. Show your work.

	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**	**Trial 5**
Distance	3.312 m	3.287 m	3.308 m	3.310 m	3.313 m

3. The lab partners were investigating the velocity of a wheeled object recoiling down a track when a compressed plunger spring is released, and they recorded their data in the table below. However, the lab instructor told them that they had made a mistake somewhere, because their calculated results did not agree with the results of the instructor's own trial.

Student Trial								
Time (s)	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**
Distance (m)	0.321	0.461	0.761	0.861	1.14	1.24	1.48	1.66
Instructor Trial								
Time	**1.00**	**2.00**	**3.00**	**4.00**	**5.00**	**6.00**	**7.00**	**8.00**
Distance (m)	0.311	0.541	0.821	0.941	1.26	1.32	1.52	1.72

Based upon the collected data shown above:

a) (*10 pts*). Calculate the average velocity of the object for **both** the student and instructor trials. Use the formula of ***v=d/t*** and show your work (including the units).

Velocity (Student) = __________________m/s Velocity (Instructor) = ___________m/s

b) (*10 pts*). Find the % difference between the instructor's calculated value and your calculated value of velocity. Show your work (including the units). (Use the instructor's value as the standard).

% diff = _____________________%

4. (*15 pts*). Plot the data from the student trial in analysis question 3 on a graph, and determine the velocity of the object by finding the slope of the line. Show your calculations here (including the units) and mark the resulting slope value below and on the graph. Ensure that you mark your name on the graph and correctly label the axes. Attach the graph to this lab report.

Slope of the Line (Velocity) = ____________________m/s

CONCLUSION (A conclusion is not required for this experiment)